Pricing an R&D Venture with Uncertain Time to Innovation

José Carlos García Franco
Onward, Inc.
jcpollo@onwardinc.com

The value of new product development projects is typically characterized by a significant amount of uncertainty. Not only do these projects face the usual risks associated with market conditions but, in addition, they face a considerable amount of uncertainty of a technical nature. This technical uncertainty often comes in two forms: (i) uncertainty over the product features resulting from the initial research and development (R&D) stages or (ii) uncertainty over the time until the R&D effort delivers a product with specific features. In this paper we will focus our attention on appraisal of projects characterized predominantly by the second form of uncertainty. The analysis presented is of great importance for firms competing for sales in established supply chains where prompt innovation times translate directly into project wins.

An important feature of our analysis is that it directly addresses the effect that managerial decisions throughout the life of the project have on its risk/reward profile and consequently on its value. Furthermore, the analysis adopts a dynamic view that allows managers to determine the sequence of value maximizing decisions as uncertainty is resolved (favorably or unfavorably) through time. In that sense, the analysis provides both a measure of the project's value (that may or may not justify its funding) and the managerial paths or decision rules that must be followed in order to realize such value. In many respects our framework formalizes, at least partially, many of the insights that seasoned managers have acquired through their business experience.

Figure 1 illustrates an important relationship between decisions and value: while our decisions flow along with time, their effect on value flows backward in time, in the sense that the value of today's decisions is determined by the value of the decisions we will make tomorrow.

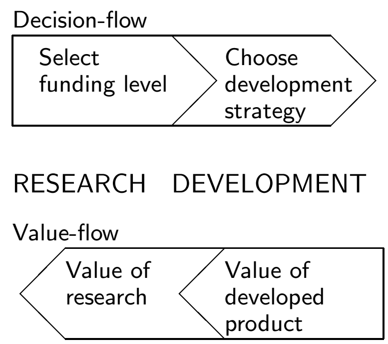

Figure 1: *Value maximization and decision matching*

Therefore, ensuring that our future decisions maximize value is critical in order to determine today's value maximizing decisions. The relationship between value and decisions will be exemplified in a simple version of an R&D case study. The case study will also illustrate a sound methodology to determine the value maximizing decision stream and the corresponding contribution of the project to the value of a firm.

1 A digital video storage appliance

Consider DigiCrate, a firm developing a new digital storage technology aimed at video applications. It is recognized that if DigiCrate is successful in developing this technology, it will provide a new standard with such capacity, speed and portability that it will enable the company to capture an important share of the stor-

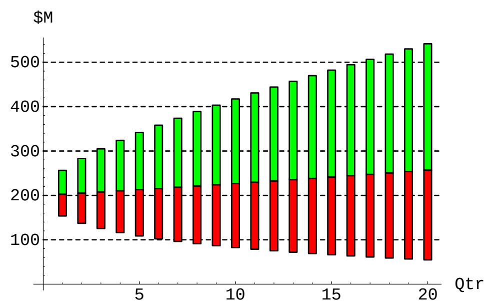

Figure 2: *Quarterly market sales forecast and volatility.*

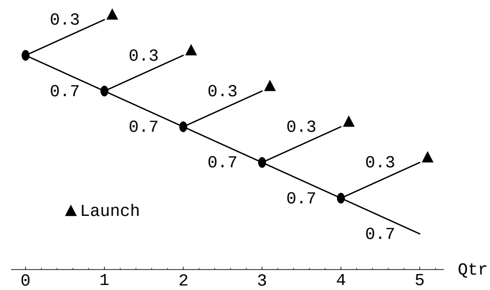

Figure 3: *Innovation arrival process.*

age market. However, the research and development effort is quite costly at a rate of \$4M per quarter. Given this significant capital outlay and the uncertainty surrounding successful development, a robust evaluation of the venture's profit potential versus costs is required in order to determine the project's economic value contribution. In the sections that follow, we will describe each of the elements that define the project's profit and cost structure and a methodology for turning our knowledge about the project into a value metric.

1.1 The market

Annual industry sales for storage technologies in DigiCrate's realm is \$800 million with an expected annual growth rate of 5%. However, sales tend to fluctuate along with market conditions, and in this particular sector exhibit an annual volatility of 35%. Figure 2 shows the corresponding forecast of the quarterly industry sales for the next 5 years. The top bars (green) extend from our expected sales forecast to the 90% best case value, while the bottom bars (red) go down to the 10% worst case value. Naturally, market size forecasts have increased variance as we predict further into the future.

If DigiCrate is successful in producing a marketable technology, it will reap a share of this market. Hence, the state of the industry sales and, more importantly, its uncertain evolution going forward, play an important role in the project's value potential.

1.2 The innovation process

In order to produce a marketable product, the research team must meet a number of transfer speed, data capacity and reliability specifications. If such specifications are not met, there is no opportunity to enter the market.

Given DigiCrate's research progress, research staff and research assets, there is a 30% chance the R&D effort produces a marketable product ready to be launched at the beginning of next quarter. If this milestone is not achieved, then the research effort may continue and there is a 30% chance the product will get launched the following quarter. If at that point in time, they still have not produced a marketable product, they can once again continue the research effort with a 30% chance of launching one quarter later. This situation continues every quarter: if no technical success has been achieved, research may continue with a 30% chance it will be achieved by the next quarter. However, recall that there is a quarterly cost of research of \$4M that may or may not be worth spending depending on market conditions. Suspending research funding at any point in time will translate into a permanent termination of the project due to high restart costs. Figure 3 illustrates this simple structure of innovation uncertainty assuming, of course, that the cost of research is paid every quarter.

1.3 Market share

Once the technology is marketable (embedded in an appliance) it will have a life-cycle of 3 years (12 quar-

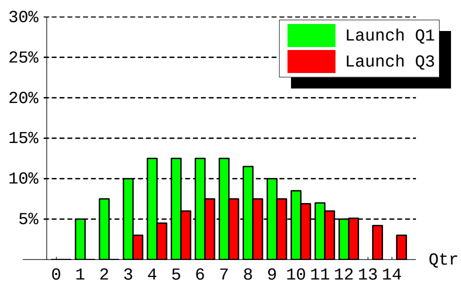

Figure 4: *Market share life-cycle.*

ters). Like most technologies, market share for our product is characterized by an adoption phase, a consolidation phase and, finally, an obsolescence phase that precedes the end of the marketable life of the product. The bars labeled `Launch Q1` in Figure 4 illustrate the market share DigiCrate would obtain if innovation is achieved in time to launch a marketable product by the next quarter.[1]

However, given market dynamics and the emergence of rival technologies, this market share profile is likely to change (probably in a detrimental manner) as our innovation time is delayed. In our case, we assume a simple structure, in which each additional quarter of delay in innovation translates into a market share reduction of 20% of the original market share. For instance, the bars labeled `Launch Q3` in Figure 4 show the market share effect of a 2 quarter launch delay. Notice, however, that we preserve the assumption that the product's life-cycle length is 3 years. We may also note that as a consequence of the assumed effect of innovation delay on market share, the research effort is worthless after 5 quarters of unsuccessful innovation. Table 3 in the appendix shows the market share profile for each (relevant) innovation scenario.

1.4 Profit margin and fixed costs

Profit margins (over sales) for this kind of technology tend to decline as we approach the obsolescence stage of the technology's life-cycle. We assumed a profit margin of 10% of sales throughout the entire life-cycle of the technology. However, any margin curve (e.g., declining margins as the technology ages) can easily be incorporated into the analysis without any major complications. In addition, we consider a quarterly fixed cost of operation of $200K. Note that this cost is in place once the product has been launched and, in some sense, replaces the $4M research cost we had prior to innovation.

2 Valuation

One fundamental question we seek to answer is: What is the value of our R&D project? Oftentimes the necessity of answering this question simply comes from the need to assess whether the project's value justifies its setup costs. Other times setup costs have been incurred already and the necessity of determining value obeys acquisition or value transfer situations. Either way, the value of the project clearly depends on both the current market and research status as well as their corresponding (uncertain) outlook. Furthermore, this value must take into consideration the flexibility managers have to alter the course of the project in the future. For instance, in order to avoid losses, managers may decide to terminate the project if the economic outlook becomes significantly unfavorable when considering the current state of research progress and market conditions. Failure to recognize this simple termination (real) option in our valuation may cause us to understate an important part of the project's profitability and miss a value creation investment opportunity.

A common building block in project appraisal is the discounted cash-flow method (DCF); however, this method is, by design, of a deterministic nature and unfit to treat uncertainty. While this shortcoming may have a marginal effect on certain investment problems, more generally, the use of DCF often becomes a source of serious errors in appraising risky investment alternatives where even little interaction between uncertainty and decisions is characteristic.

In contrast to DCF, the valuation approach described here delivers not only a more accurate value measure, but also a sequence of value maximizing decisions as the realization of uncertain events unfolds throughout

[1] We assume, for illustration simplicity, a deterministic market share. However, the addition of uncertainty to this forecast is easily accommodated by our analytic framework.

the life of the project.

2.1 The value of innovation scenarios

Let us define some useful notation:

- I_k, industry sales in quarter k.
- S_k, DigiCrate's market share in quarter k.
- P_k, profit margin (over sales) in quarter k.

Suppose DigiCrate achieves technical success and is ready to launch by the next quarter. The quarterly cash-flow C_k is defined by the difference between profit over sales and fixed costs, that is,

$$C_k = I_k \times S_k \times P_k - \$200,000.$$

Recall that the life-cycle of our product was 3 years (12 quarters); assuming (for simplicity) a constant risk-free rate, the present value of our technology after innovation is given by

$$V = \sum_{k=1}^{12} \frac{C_k}{(R)^k}$$

where R is the quarterly risk-free return. We assume an annual risk-free rate of 4%, which with continuous compounding yields $R = 1.01005$.[2]

Note that this expression assumes that we market the product until the end of its life-cycle, that is, it ignores the option of early withdrawal of the product from the marketplace.[3] Notice also that we are discounting all cash-flows at the risk-free rate, a situation which may seem inappropriate at first given the significant risk imposed by market sales uncertainty. The next section will throw some light on issues regarding risk discounting.

2.1.1 Risk

In order to appropriately discount a risky cash-flow, it is necessary to decompose the cash-flow into its different components. Recall that the quarterly cash-flow is given by

$$C_k = I_k \times S_k \times P_k - \$200,000.$$

The \$200,000 fixed operating cost is not uncertain; hence, it is appropriate to discount it at the risk-free rate just like we would do with any deterministic cash flow (e.g., an annuity, or a bond). The profit over sales, however, is uncertain and likely to be related[4] in some fashion to assets in the financial markets (e.g., a basket of stocks from companies in the digital storage sector). If this is the case, prices in the financial markets give us information about how the market (as an abstract, yet almighty and on occasion ruthless entity) discounts this particular kind of risk. Using market prices and correlation information, we can arrive at the appropriate risk adjustment required by our forecasts/expectations of our uncertain, yet market related, cash-flows. Note that in our analysis only industry sales require a risk adjustment as we adopted the view that margins and market share are predictable enough to be assumed deterministic.

Table 2 in Appendix A shows our expected market forecasts and their corresponding risk adjustment. We used the extreme assumption that sales are perfectly correlated to a financial market asset. Hence, our risk adjustment consists of simply deflating (discounting) the forecast by the expected rate above risk-free that similar market priced assets require. The details of risk discounting are omitted here, but the important lessons are: (1) that since deterministic cash-flows are invariant to market conditions, they do not require any risk adjustment and (2) that cash-flows which are correlated with the financial markets must be risk adjusted usually by deflating their growth rate in a way that is consistent with related financial securities. Hence, different cash-flow components must be discounted at different rates. Fortunately, once all cash-flows with market risk have been adjusted, we can treat them the same as risk-free cash-flows and discount all of them at the risk-free rate. The reader may want to refer to the Fall 1998 Investment Science newsletter for more details on risk adjustment and correlation pricing.

[2] This number comes from $\exp[.04/4] = 1.01005$, the result under, say, quarterly compounding is not much different: $1 + .04/4 = 1.01$.

[3] If a product is becomes unprofitable, it may be wise to remove it from the shelves.

[4] Statistically correlated.

τ	V_τ	Cost	Net	Pr
1	\$20.65M	4.M	16.65M	0.3
2	\$16.09M	7.96M	8.13M	0.21
3	\$11.53M	11.88M	-0.35M	0.147
4	\$6.98M	15.76M	-8.79M	0.1029
5	\$2.42M	19.61M	-17.19M	0.07203
≥ 6	\$0.M	19.61M	-19.61M	0.16807

Table 1: Value and probability of innovation scenarios.

2.1.2 The value of delayed innovation

Now suppose that innovation is not achieved by next quarter, but rather, that we are not ready to launch our product until τ quarters from now. The value for time τ innovation is

$$V_\tau = \sum_{k=\tau}^{\tau+11} \frac{C_k}{(R)^k}$$

where the cash-flows C_k account for the reduction in market share induced by the delay in innovation as indicated in Table 3 of Appendix A and where sales have been appropriately risk adjusted.

Table 1 shows project values assuming different innovation scenarios. Each scenario corresponds to a different innovation time, starting with product release next quarter all the way to 5 quarters from now. Recall that there is no value for innovation occurring later than 5 quarters from now.

2.2 Overall project value

We have determined the value of different innovation scenarios, we can also compute the present value of the funding required under each scenario and the corresponding net present value. Furthermore, we can compute the probability of each scenario by simply following the events described in Figure 3. For instance, the probability that innovation occurs one quarter from now is simply 0.30, assuming, of course, that research is funded (at a cost of \$4M). The probability that innovation occurs two quarters from now, is simply the probability we fail the first period multiplied by the probability that we are successful the following period, that is, $\text{Prob}(\tau = 2) = 0.7 \times 0.3 = 0.21$, again assuming the funding cost of \$4M is covered each period. We can proceed in a similar way in order to obtain the probabilities of each of our 5 relevant innovation scenarios. These probabilities are shown in the last column of Table 1.

Having scenario net present values and probabilities, one may be tempted to just take the expected value (a probability weighted average of scenario values) in order to appraise the project. Such calculation yields a value of \$1.21M for our innovation project. This value accounts for all expenditures to be made during the life of the project and may be regarded as the economic contribution of the project. This is almost correct. However, we must account for the fact that we may decide to terminate the project if the profit outlook is not sufficiently favorable to justify future research funding. From the data in Table 1 it is apparent that it is better to terminate the project if the project has not delivered a marketable product in the first couple of quarters. This is a reasonable way to think, however, given the volatility of industry sales, it may be profitable to continue research funding if the market is "booming", if it is not then we simply terminate the project and avoid greater losses. Furthermore, while the values from Table 1 come from adequately risk adjusted sales forecasts, such forecasts are ill-suited for "down the road" decision making as it is a certainty that evolution of uncertainty will not follow them. The next section will illustrate a procedure that will allow us to properly account for the early termination option that is naturally present in our project.

2.3 The early termination option

In order to evaluate a project abandonment decision we must know: (i) the value of abandonment and (ii) the value of continuing project funding taking into consideration the value of all future decisions. In that sense our analysis must proceed backwards in time, that is, we must consider the backwards value flow we previously discussed at the beginning of this case study.

Period 4 decision rule

Suppose no innovation has been achieved after four quarters, from this point forward our alternatives are quite simple. We can (i) fund research for one more quarter and take a last shot at launching a profitable product (recall that innovation after 5 quarters is worthless), or (ii) call it quits and take our loss. Let $F_5(4)$ denote the value (in period 4 dollars) of successful innovation by the fifth quarter. To be precise,

$$F_5(4) = \sum_{k=5}^{16} \frac{C_k}{(R)^{k-4}}.$$

The probability at Q4 of innovating by Q5 is 30%, so given Q4 market conditions (i.e. the current value of industry sales I_4), the optimal value of the project $V_4^*(I_4)$ is given by the maximum of the termination value of zero and the expected continuation value, that is,

$$V_4^*(I_4) = \left(0.30 \times \mathrm{E}\left[F_5(4)|I_4\right] - \$4\mathrm{M}\right)^+ \qquad (1)$$

where $\mathrm{E}[\cdot|I_4]$ denotes the expectation conditional on the (industry sales) information available at quarter 4 and where $(x)^+ \equiv \max\{x, 0\}$.

Correspondingly, we can determine a decision rule that ponders whether the project's remaining potential justifies the research expense. In particular, our decision rule takes the form

- Fund research if

$$0.30 \times \mathrm{E}[F_5(4)|I_4] > \$4\mathrm{M}. \qquad (2)$$

- Terminate the project if

$$0.30 \times \mathrm{E}[F_5(4)|I_4] \leq \$4\mathrm{M}. \qquad (3)$$

Market sales in Q4 give us an indication of what future sales look like and therefore determine the expected value of Q5 innovation. Recall that the (uncertain) evolution of market sales I_k is the only source of market uncertainty in our project. Some computation allows us to write

$$\mathrm{E}[F_5(4)|I_4] = -2.2503 + 0.0229\, I_4 \qquad (4)$$

Appendix B briefly explains how to compute the expectation in (4). However, our emphasis on this case study is not on the computational details, but rather on the significance of the variables at play (and of course the acknowledgment that they can be computed).

Equation (3) together with (4) implies the following decision rule:

- Fund research if

$$I_4 > \$680.51\mathrm{M} \qquad (5)$$

- Terminate the project if

$$I_4 \leq \$680.51\mathrm{M} \qquad (6)$$

We now know how we should manage the flexibility to terminate research funding if we have reached Q4 with no innovation.

Period 3 decision rule

We must now determine what our management policy is for the same situation in Q3. Suppose we are at Q3. If we terminate the project we trivially get zero value. If, however, we fund the project there is a 30% chance we achieve technical success by Q4 in which case the value of the project (given Q3 industry sales) is $\mathrm{E}_3[F_4]$ where F_4 is the present value of the cash-flow generated by Q4 innovation (see Table 4). On the other hand if technical success is not achieved by Q4, we must then decide whether to terminate the project or keep funding it. To this effect, we rely in the optimal decision rule derived for Q4. Therefore the optimal Q3 value of our project is given by

$$V_3^*(I_3) = \left(0.3\,\mathrm{E}[F_4|I_3] + 0.7\,\frac{\mathrm{E}[V_4^*(I_4)|I_3]}{R} - \$4\mathrm{M}\right)^+.$$

where R is the quarterly risk-free return.

Correspondingly, given the Q3 sales volume I_3, the Q3 decision rule is:

- Fund research if

$$0.3 \times \mathrm{E}[F_4|I_3] + 0.7 \times \frac{\mathrm{E}[V_4^*(I_4)|I_3]}{R} - \$4\mathrm{M} > 0$$

- Terminate the project if

$$0.3 \times \mathrm{E}[F_4|I_3] + 0.7 \times \frac{\mathrm{E}[V_4^*(I_4)|I_3]}{R} - \$4\mathrm{M} \leq 0,$$

Implicit in this rule, there is a critical value of Q3 industry sales I_3 that satisfies

$$0.3 \times \mathrm{E}[F_4|I_3] + 0.7 \times \frac{\mathrm{E}[V_4^*(I_4)|I_3]}{R} - \$4\mathrm{M} = 0.$$

Earlier periods

Following the same logic we used for period 3, we arrive to the relationship

$$V_k^*(I_k) = \left(0.3\,\mathrm{E}[F_{k+1}|I_k] + 0.7\,\frac{\mathrm{E}[V_{k+1}^*(I_{k+1})|I_k]}{R} - \$4\mathrm{M}\right)^+,$$

and the corresponding period k decision rule:

- Fund research if

$$0.3\,\mathrm{E}[F_{k+1}|I_k] + 0.7\,\frac{\mathrm{E}[V_{k+1}^*(I_{k+1})|I_k]}{R} - \$4\mathrm{M} > 0.$$

- Terminate project if

$$0.3\,\mathrm{E}[F_{k+1}|I_k] + 0.7\,\frac{\mathrm{E}[V_{k+1}^*(I_{k+1})|I_k]}{R} - \$4\mathrm{M} \leq 0.$$

These results allow us to evaluate the project by recursively finding the project termination boundaries and the corresponding project value. Note that this evaluation must be done in a "backwards" fashion, working our way from the last relevant period in our analysis towards the present value of the project. The computational requirements of this evaluation are admittedly more complex that present value analyses, yet, the required methods are within the reach of modern financial engines and Real Options Calculators that may be added to spreadsheet programs. Appendix C shows some of required computational procedures used in this case.

Figure 5 shows the project termination critical values for our case study. These values indicate for each period, the minimum level of industry sales that justify continuation of research funding in the case the research has not been successful already in delivering a marketable product. As it is to be expected, in the absence of successful innovation, as time goes by, we require higher industry sales in order to justify funding to the point where innovation later than Q5 is worthless and unable to justify research spending past Q4.

The project termination policy is an extremely valuable output of our analysis, as it delivers a value maximizing project management policy. More importantly, it provides a decision tool that indicates the value maximizing course of action throughout the life of the project. An extremely valuable tool, not just in terms of appraisal but also in terms of project management guidance.

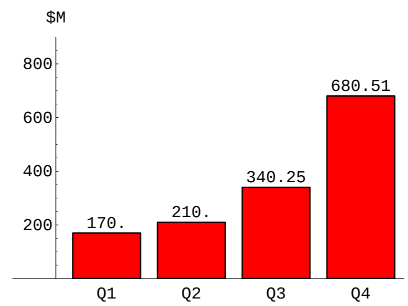

Figure 5: *Industry sales critical values for project termination.*

One thing to notice is that given the current quarterly industry sales of \$200M, it is always prudent to fund research at least for the initial quarter. From then on, fluctuations in industry sales along with our funding termination policy will determine for how long the project will be funded, unless, of course, innovation is achieved and the product is marketed. Under these circumstances, the value of our project is \$3.04 million. That is, given current industry sales of \$200 million (per quarter)[5] we have that $V_0^*(\$200\mathrm{M}) = \$3.04\mathrm{M}.$

3 Conclusion

This case study explored the interplay between different kinds of risk typical of R&D projects and allowed us to uncover how their interaction determines project value. As a starting point we identified in a clear manner the nature of both technical or innovation risk and market risk. The former came in the form of the probability each quarter that the firm's research is able to produce a marketable product by the next quarter given that it has not yet achieved such goal. This set of probabilities defined the *innovation process*. So in a sense the innovation process defined the opportunity of entering a market, which, of course, came at a cost given by the

[5]We distribute the annual sales volume of \$800M equally among quarters.

funding needs of the research effort each quarter. In addition to innovation risk, the project also faced market risk in the sense that, in the event of being able to market a product, its profitability is uncertain and directly related to overall industry sales. Industry sales are a market variable and, as opposed to technical risk, may be risk adjusted using the information of related variables in the financial markets. In our case we made matters simple via the assumption that there was a perfect financial asset that served as a proxy for industry sales. This assumption allows for a simple risk adjustment by discounting by the excess rate of return with respect to the risk-free rate[6]. The analysis allowed us to obtain a probabilistic description of the value of the possible innovation scenarios which properly averaged led to a project value of $1.21M.

Up to this point, our analysis ignored the managerial flexibility usually found in this kind of projects. As an example of managerial flexibility, we incorporated the option to stop project funding and cancel the project if the profit outlook is not sufficient to justify continued funding of research. This flexibility inevitably increases the value of the project since it allows us to avoid potential losses in unfavorable situations. The analytic challenge lies in determining a policy that explicitly states when and under what circumstances the option to terminate the project should be exercised. Furthermore, the numerical analysis required is relatively complex. In the second part of this case, we explicitly illustrated the appropriate analytic procedure and provided the corresponding results based on its numerical evaluation. The result was an optimal termination policy that indicates the value maximizing set of actions throughout the life of the project. Correspondingly, we were able to determine the value of an optimally managed project which increased to $3.04M from the $1.21M obtained in the first part of our analysis. This is quite a dramatic change in appraisal and it illustrates the point that an appraisal methodology that accounts for "down the road" managerial flexibility can uncover significant value in investment projects. In our case the project value increased by 150%, an increase that would have remain hidden using traditional DCF methodologies and that may have led to sub-optimal budgeting or acquisition decisions.

[6]Given a model for industry sales with geometric growth and constant volatility. Please see the Fall 1998 Investment Science Newsletter for details

This case illustrated the use of modern appraisal methodologies (real options) in R&D applications where time to innovation is uncertain. The purpose: (1) to numerically illustrate the usefulness of a modern project evaluation approach in this context., and (2) to illustrate the procedure and way of thinking required to carry out the analysis. While computation is a bit complicated and may need real option solvers or specialized algorithms, the sole way of thinking often leads to better evaluation. For instance, in our case, an analyst equipped only with a spreadsheet may uncover value by imposing a policy that terminates the project if technical success is not achieved by Q2. Even with this simple policy, the analyst will uncover close to 50% additional value in the project with respect with a DCF analysis without a termination policy. This is far from the 150% obtained by computing an optimal policy that responds to market conditions, but at least it provides a good starting point for measuring value provided one recognizes it is better than DCF and in all probability not better than an analysis with an optimal policy search.

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A Sales and Market share tables

	Sales Forecast ($M)	Risk Adjusted ($M)		Sales Forecast ($M)	Risk Adjusted ($M)
Q1	202.52	202.01	Q9	223.81	218.83
Q2	205.06	204.04	Q10	226.63	221.03
Q3	207.64	206.09	Q11	229.48	223.26
Q4	210.25	208.16	Q12	232.37	225.50
Q5	212.90	210.25	Q13	235.29	227.77
Q6	215.58	212.37	Q14	238.25	230.05
Q7	218.29	214.50	Q15	241.25	232.37
Q8	221.03	216.66	Q16	244.28	234.70

Table 2: *Industry sales forecast.*

Launch Q1

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
05.00%	07.50%	10.00%	12.50%	12.50%	12.50%	12.50%	11.50%	10.00%	08.50%	07.00%	05.00%

Launch Q2

Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13
04.00%	06.00%	08.00%	10.00%	10.00%	10.00%	10.00%	09.20%	08.00%	06.80%	05.60%	04.00%

Launch Q3

Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14
03.00%	04.50%	06.00%	07.50%	07.50%	07.50%	07.50%	06.90%	06.00%	05.10%	04.20%	03.00%

Launch Q4

Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
02.00%	03.00%	04.00%	05.00%	05.00%	05.00%	05.00%	04.60%	04.00%	03.40%	02.80%	02.00%

Launch Q5

Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15	Q16
01.00%	01.50%	02.00%	02.50%	02.50%	02.50%	02.50%	02.30%	02.00%	01.70%	01.40%	01.00%

Table 3: *Market share scenarios.*

τ	$\mathrm{E}[F_\tau(\tau-1)\|I_{\tau-1}]$
1	$-2.2503 + 0.1145\, I_0/(10^6)$
2	$-2.2503 + 0.0916\, I_1/(10^6)$
3	$-2.2503 + 0.0687\, I_2/(10^6)$
4	$-2.2503 + 0.0458\, I_3/(10^6)$
5	$-2.2503 + 0.0229\, I_4/(10^6)$

Table 4: Innovation time τ conditional expected values (units in millions of dollars).

B Innovation Conditional Expected Value

Let R denote the (constant by assumption) quarterly risk-free rate of return. The time $k-1$ value of innovation at time k is given by

$$\begin{aligned} F_k(k-1) &= \sum_{i=k}^{k+11} \frac{C_i}{R^{i-(k-1)}} \\ &= \sum_{i=k}^{k+11} \frac{I_i \times S_i \times P_i - \$200,000}{R^{i-(k-1)}} \end{aligned}$$

where S_i, P_i are the market share and profit margin profiles corresponding to date k innovations (i.e., product launch starting in quarter k).

Let $\hat{\mathrm{E}}$ denote the risk-adjusted expectation operator, that is, an expectation that adequately adjusts the expected value of market variables. Then, given the uncertainty assumptions[7] over market sales, we have

$$\hat{\mathrm{E}}[I_{k+s}|I_k] = I_k\, R^s.$$

Hence, we can write the conditional expected value of F_k as

$$\begin{aligned} \hat{\mathrm{E}}[F_k(k-1)|I_{k-1}] &= \hat{\mathrm{E}}\left[\sum_{i=k}^{k+11} \frac{C_i}{R^{i-(k-1)}} \middle| I_{k-1}\right] \\ &= \sum_{i=k}^{k+11} \frac{\hat{\mathrm{E}}[I_i|I_{k-1}]\, S_i\, P_i - \$200,000}{R^{i-(k-1)}} \\ &= \sum_{i=k}^{k+11} \frac{I_{k-1}\, R^{i-(k-1)}\, S_i\, P_i}{R^{i-(k-1)}} - \sum_{i=k}^{k+11} \frac{\$200,000}{R^{i-(k-1)}} \\ &= I_{k-1} \sum_{i=k}^{k+11} (S_i\, P_i) - \sum_{i=k}^{k+11} \frac{\$200,000}{R^{i-(k-1)}}. \end{aligned}$$

Table 4 shows the corresponding values of the innovation conditional project values for all relevant innovation times in this case study.

[7] Perfect correlation of sales to some marketed asset and a geometric growth model.

C Optimal value to go

In this appendix we show the derivation of the optimal value to go and the corresponding optimal exercise thresholds. All values are expressed in millions of dollars.

Let C denote the quarterly cost of research, then we have relationship

$$V_k^*(I_k) = \max\left\{0, 0.3\,\mathrm{E}[F_{k+1}(k)|I_k] + 0.7\,\frac{1}{R}\mathrm{E}[V_{k+1}^*(I_{k+1})|I_k] - C\right\}. \tag{7}$$

Quarter 4 decision and value

For Q4 we have

$$\begin{aligned} V_4^*(I_4) &= \max\{0, 0.3\,\mathrm{E}[F_5(4)|I_4] - 4\} \\ &= \max\{0, 0.3\,(a_5 + b_5\,I_4) - 4\} \\ &= \max\{0, 0.3\,a_5 + 0.3\,b_5\,I_4 - 4\} \end{aligned}$$

with $a_5 = -2.2503$ and $b_5 = 0.0229$.

Let $k_4 = (4 - 0.3\,a_5)/(0.3\,b_5) = 680.51$, then we have

$$V_4^*(I_4) = \begin{cases} 0 & \text{if} \quad I_4 \le k_4 \\ 0.3\,a_5 + 0.3\,b_5\,I_4 - 4 & \text{if} \quad I_4 > k_4. \end{cases}$$

Quarter 3 decision and value

For Q3 we have

$$\begin{aligned} V_3^*(I_3) &= \max\left\{0, 0.3\,\mathrm{E}[F_4(3)|I_3] + 0.7\,\frac{1}{R}\,\mathrm{E}[V_4^*(I_4)|I_3] - 4\right\} \\ &= \max\left\{0, 0.3\,(a_4 + b_4\,I_4) + 0.7\,\frac{1}{R}\,\mathrm{E}[V_4^*(I_4)|I_3] - 4\right\} \end{aligned}$$

where $a_4 = -2.2503$ and $b_4 = 0.0458$.

The evaluation of $\mathrm{E}[V_4^*(I_4)|I_3]$ is often complicated, however, in this case we are still able to do it in a relatively simple way. Let $g(x)$ be the probability density function (PDF) of I_4 conditional on I_3. The geometric growth model[8] for industry sales assumes I_4 is log-normally distributed. Specifically, we have that $\log I_4$ is normally distributed with mean $\log I_3 + (r + .5\sigma^2)/4$ and variance $\sigma^2/4$. Let $f(x)$ denote the PDF of $\log I_4$ given I_3 and $F(x)$ the corresponding cumulative distribution function (CDF), then we have

[8] Specifically, we used Geometric Brownian Motion, a stochastic process which is the building block for most models of continuous-time finance.

$$
\begin{aligned}
\mathrm{E}[V_4^*(I_4)|I_3] &= \mathrm{E}\left[(0.3\,a_5 + 0.3\,b_5\,I_4 - C)^+ \Big| I_3\right] \\
&= \int_{k_4}^{\infty} (0.3\,a_5 + 0.3\,b_5\,x - C)\ g(x)\,dx \\
&= (0.3\,a_5 - C)\int_{\log k_4}^{\infty} f(x)dx + 0.3\,b_5 \int_{k_4}^{\infty} x\,g(x)\,dx \\
&= (0.3\,a_5 - C)\,(1 - F(\log_k)) +\ 0.3\,b_5 \int_{\log k_4}^{\infty} \exp[x]\,f(x)\,dx.
\end{aligned}
$$

Let $\nu = \log I_3 + (r + .5\sigma^2)/4$ and $\varsigma^2 = \sigma^2/4$, then we have

$$
f(x) = \frac{1}{\sqrt{2\pi}\varsigma}\exp\left[-\frac{(x-\nu)^2}{2\varsigma^2}\right],
$$

and we can write

$$
\begin{aligned}
0.3\,b_5 \int_{\log k_4}^{\infty} \exp[x]\,f(x)\,dx &= 0.3\,b_5 \frac{1}{\sqrt{2\pi}\varsigma}\int_{\log k}^{\infty} \exp\left[x - \frac{(x-\nu)^2}{2\varsigma^2}\right]\,dx \\
&= 0.3\,b_5\,\frac{1}{\sqrt{2\pi}\varsigma}\int_{\log k_4}^{\infty} \exp\left[-\frac{\big(x-(\nu+\varsigma^2)\big)^2 + \nu^2 - \big(\nu+\varsigma^2\big)^2}{2\varsigma^2}\right]\,dx \\
&= 0.3\,b_5\,\frac{1}{\sqrt{2\pi}\varsigma}\exp\left[\frac{(\nu+\varsigma^2)^2 - \nu^2}{2\varsigma^2}\right]\int_{\log k_4}^{\infty} \exp\left[-\frac{(x-(\nu+\varsigma^2))^2}{2\varsigma^2}\right]\,dx \\
&= 0.3\,b_5\ \exp\left[\frac{(\nu+\varsigma^2)^2 - \nu^2}{2\varsigma^2}\right]\ \left(1 - \phi\left[\frac{\log k_4 - (\nu+\varsigma^2)}{\varsigma}\right]\right).
\end{aligned}
$$

where $\phi[\cdot]$ is the standard normal distribution function, that is, $\phi(a) = \mathrm{Prob}(\epsilon \le a)$ where ϵ is a standard normal random variable.

Using the above results we can evaluate $V_3^*(I_3)$ and solve for the optimal decision boundary $k_3 = 340.25$. Therefore, we have

$$
V_3^*(I_3) = \begin{cases} 0, 0.3\,(a_4 + b_4\,I_4) + 0.7\,\frac{1}{R}\,\mathrm{E}[V_4^*(I_4)|I_3] - 4 & \text{if} \quad I_3 > k_3 \\ 0 & \text{if} \quad I_3 \le k_3. \end{cases}
$$

Earlier periods

The decision boundaries k_i, $i = 0, 1, 2$ and the corresponding value functions $V_i^*(I_i)$ where estimated in a similar fashion using the relationship

$$
V_k^*(I_k) = \max\left\{0, 0.3\,\mathrm{E}[F_{k+1}(k)|I_k] + 0.7\,\frac{1}{R}\mathrm{E}[V_{k+1}^*(I_{k+1})|I_k] - C\right\}. \tag{8}
$$

However, numerical approximation techniques where utilized as there is no easy way to obtain analytic solutions to compute the required expected values. Nevertheless, they rely on the same recursive relationship (8). The results are reported on Figure 5.